Empirical growth models with spatial effects*

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Abstract. Recent contributions to the regional science literature have considered spatial effects in empirical growth specifications. In the case of spatial dependence, following theoretical arguments from new economic geography, and endogenous growth models, this phenomenon has been associated with the existence of externalities that cross regional borders. However, despite the general consensus that interactions or externalities are likely to be the major source of spatial dependence, they have been modelled in a rather *ad hoc* manner in most existing empirical studies. In contrast, we advocate basing the analysis on structural growth models which include externalities across economies, applying the appropriate spatial econometrics tools to test for their presence and estimate the magnitude of these externalities in the real world.

JEL classification: O40, R11, R12

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1 Introduction

Economic growth is a topic that has, for a long time, attracted the attention of economists, more so in recent decades. Theoretical contributions have emphasised

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the role of different factors in determining the steady state level of income per capita, and in promoting growth. In addition, there is an enormous amount of empirical evidence relating to the assumptions and predictions from theoretical models, with a large number of empirical contributions focussing on regional economic growth (Neven and Gouyette 1995; Sala-i-Martin 1996; Carlino and Mills 1993, 1996; Bernard and Jones 1996; Chatterji and Dewhurst 1996; Armstrong 1995). One interesting characteristic of these analyses is that, as in the case of heterogeneous countries, regions have been considered isolated economies. In other words, empirical specifications almost invariably exclude interactions across regions. However, theoretical and empirical arguments suggest that regions, as well as not being homogeneous, are also not independent. For instance, Rey and Montouri (1999, p. 144) indicate that "Despite the fact that theoretical mechanisms of technology diffusion, factor mobility and transfer payments that are argued to drive the regional convergence phenomenon have explicit geographical components, the role of spatial effects in regional studies has been virtually ignored".

The problem with aspatial empirical analyses that have ignored the influence of spatial location on the process of growth is that they may have produced biased results, and hence misleading conclusions. To address this problem, some regional economists and economic geographers suggest accommodating spatial heterogeneity and dependence in regional growth specifications (Amstrong 1995; Rey and Montouri 1999; López-Bazo et al. 1999, Bivand and Brundstad 2006). Their suggestions are broadly consistent with assumptions and predictions related to endogenous growth theory and new economic geography models, which stress the role of interactions across agents that, for instance, cause economic activity to agglomerate in some areas and not in others (Fujita et al. 1999). External effects are supposed to be linked to the size of the market, to access to specialised services, to forward and backward linkages, to knowledge diffusion and to similar norms, institutions and policies across different regions. Put very simply, if we assume that firms are heterogeneous and always interacting with each other, then the fact that they are often located in different regions will cause regions to be heterogeneous and interdependent.

Bernat (1996) and Rey and Montouri (1999) were among the first to specifically include spatial effects in empirical growth exercises (see also Fingleton and McCombie 1998; Fingleton 1999; López-Bazo et al. 1999). Bernat (1996), for example, tested the simplest version of the so-called Kaldor's Laws in the set of US States, controlling for spatial dependence. Likewise Rey and Montouri (1999), coming from a neoclassical perspective rather than a heterodox perspective, checked for absolute β-convergence under spatial heterogeneity and spatial dependence. These early analyses precipitated a series of studies explicitly including spatial effects in growth specifications, mainly in the form of the spatial error model and the spatial lag model, although there has also been some estimation of the spatial cross-regressive model (see Anselin 1988, for a description). The selection of one of these models is almost invariably based on a statistical criterion, basically the one proposed in Anselin and Rey (1991), and in Florax and Folmer (1992). Hence, despite the broad agreement that interactions or externalities across regions are likely to be the major source of spatial dependence, they have been modelled in a rather *ad hoc* manner in most of the existing empirical studies. What is more surprising is that the empirical evidence on the preferred spatial specification is mixed, and seems to depend on the set of regions, time period, specification, etc. (Armstrong 1995; Bernat 1996; Rey and Montouri 1999; Pons-Novell and Viladecans 1999; Vayá and Moreno 2002; Niebuhr 2001; Kosfeld et al. 2002; Le Gallo et al. 2003; Arbia et al. 2003; Ying 2003; Fingleton 2001, 2004; Dall'erba and Le Gallo 2005; Rey and Janikas 2005 and Abreu et al. 2005 survey the existing evidence).

The question of the correct specification is a very important one, since it turns out that each spatial specification (substantive or nuisance) produces rather different interpretations and policy implications for the process of economic growth. Using the words by Bernat (1996, p. 466) in the case of the spatial error model, "a region's growth is affected by growth in neighbouring regions only to the extent that neighbouring regions have above or below normal growth", while for the spatial lag model "a region's growth is directly affected by growth in neighbouring regions, and this effect is independent of the effect of the exogenous variables".[1] In the words of Rey and Montouri (1999, p. 150 and 153), the reasoning for the spatial error model has to do with the fact that "movements away from some steady state equilibrium may not be a function of region-specific shocks alone, but instead (. . .) of a complex set of shock spillovers", whereas in the spatial lag specification the "growth rate in a region may relate to those in its surrounding regions after conditioning on the starting year levels of income".

The assumption in this article is that externalities across regions in long-run growth is mostly a substantive phenomenon caused by technological diffusion and pecuniary externalities, while the regional transmission of random shocks only plays a minor role in the process of growth in the long-run.[2] Accordingly, spatial dependence in empirical growth models should be of the substantive type (spatial lag and/or spatial cross-regressive). The preference for the nuisance case (spatial error) in a large number of studies is the result of the failure of standard spatial econometrics tools to detect the true externality mechanisms, especially when the growth model is underspecified. In contrast with the *ad hoc* method applied in most of the literature so far, we base the analysis on a structural growth model including externalities across economies, and apply the appropriate spatial econometrics tools to test for their presence and estimate their magnitude in the real world. This is basically the approach used in recent contributions by Fingleton (2001 and 2004), López-Bazo et al. (2004), and Egger and Pfaffermayr (2006).

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[1] In fact, it depends on the indirect effects of the exogenous variables plus the indirect effects of the shocks, as determined by the Leontief expansion of the spatial lag specification. This is discussed more fully below in the context of equation (3).

[2] As suggested by two referees, the interpretation of random shocks in a cross section of growth rates averaged over several years is not easy. They might be more closely related to unobserved determinants and measurement errors that are correlated across regions than to, for instance, shocks originating from business cycles. Actually, the nature of spatial autocorrelation may, to some extent, also depend on many factors such as the choice of the weight matrix, the presence of spatial heterogeneity and the aggregation level of the units of observation.

The rest of this article tries to illustrate these points for the case of the Baumol/Barro/Mankiw et al. equation, and for the so-called Verdoorn's Law equation (Kaldor's second Law).

The structure of this article is as follows. The next section briefly describes empirical growth specifications with substantive and nuisance spatial dependence. It also discusses the type of externalities related to the resulting specifications. Section 3 presents the major characteristics of two growth models that include externalities across regions caused by technological diffusion. It shows the similarity between their empirical specifications and the Durbin representation of the spatial error model when no control variables are included in the regression. The empirical evidence illustrating that the *ad hoc* application of the spatial econometrics selection method procedures can provoke misleading conclusions about the type of spatial externalities, is presented in Sect. 4. Finally, Sect. 5 concludes.

2 Empirical growth models and spatial dependence

There are two traditional specifications that have been extensively used in the literature to analyse regional growth. The first one is the renowned convergence equation that, with some minor variations, was derived and applied in the seminal papers of Baumol (1986), Barro and Sala-i-Martin (1992) and Mankiw et al. (1992). The second is the specification linked to Verdoorn's Law that relates growth in labour productivity to growth of output in the manufacturing sector (for evidence using samples of regions, see Harris and Lau 1998; Fingleton and McCombie 1998; León-Ledesma 2000). As mentioned above, a growing number of contributions have accounted for spatial dependence in both specifications.

2.1 Spatial dependence in the convergence equation

In the case of the convergence equation, growth in a region over a given period ($\mathbf{g_y}$) is inversely related to its initial income per capita ($\mathbf{y_0}$) as a result of the mechanism of convergence towards its steady state caused by decreasing returns to capital accumulation. Additional variables in the specification ($\mathbf{X}$) control for factors determining differences in the steady states across regions. The resulting specification is of the following form:

$$\mathbf{g_y} = \mathbf{c} - \left(1 - e^{-\beta T}\right) \ln(\mathbf{y_0}) + \mathbf{X}\boldsymbol{\delta} + \boldsymbol{\varepsilon} \qquad (1)$$

where $\mathbf{c}$ denotes the intercept, $\boldsymbol{\varepsilon}$ a well-behaved error term and the scalar β is the measure of the speed of convergence. When $\beta > 0$ and significant, and $\boldsymbol{\delta}$ is a vector whose elements are non-significant, we conclude in favour of absolute β-convergence, while in the case of β significantly greater than zero, and $\boldsymbol{\delta}$ a significant vector of coefficients, the outcome is conditional β-convergence.

Spatial versions of the convergence equation include the spatial lag of growth rates (spatial lag model), a spatial structure in the perturbance (spatial error model) or the spatial lag of the initial income per capita (spatial cross-regressive model). In brief, the expression for the spatial lag convergence equation is:

$$\mathbf{g}_{\mathrm{y}} = \mathbf{c} - \left(1 - \mathrm{e}^{-\beta T}\right)\ln\left(\mathbf{y}_{\mathbf{0}}\right) + \mathbf{X}\boldsymbol{\delta} + \gamma\mathbf{W}\mathbf{g}_{\mathrm{y}} + \boldsymbol{\varepsilon} \tag{2}$$

where $\mathbf{Wg}_{\mathrm{y}}$, the spatial lag of growth rates, is obtained by premultiplying the vector of regional growth rates by the so-called spatial weights matrix, $\mathbf{W}$. This matrix determines the interactions across regions.

The spatial lag specification in (2) includes the fact that growth in each region is potentially affected by growth in its neighbouring regions. In addition, we can rewrite (2) as:

$$\begin{aligned} \mathbf{g}_{\mathrm{y}} &= (\mathbf{I} - \gamma\mathbf{W})^{-1}\left[\mathbf{c} - \left(1 - \mathrm{e}^{-\beta T}\right)\ln\left(\mathbf{y}_{\mathbf{0}}\right) + \mathbf{X}\boldsymbol{\delta} + \boldsymbol{\varepsilon}\right] \\ \mathbf{g}_{\mathrm{y}} &= (\mathbf{I} - \gamma\mathbf{W})^{-1}[\mathbf{Zb} + \boldsymbol{\varepsilon}] = (\mathbf{I} - \gamma\mathbf{W})^{-1}\mathbf{Zb} + (\mathbf{I} - \gamma\mathbf{W})^{-1}\boldsymbol{\varepsilon}, \end{aligned} \tag{3}$$

Which we have also given in generic form, with $\mathbf{Z}$ equal to the matrix of variables with columns equal to the constant, $\ln(\mathbf{y}_0)$, and the set of conditioning variables $\mathbf{X}$. Following the typology of spatial externalities introduced by Anselin (2003), we can associate the structure in (3) with the presence of global externalities in the growth process. Growth in each region is not only affected by its own initial per capita income and its conditioning variables, but also by the magnitudes of these variables in the whole system of regions. Depending on the structure of the $\mathbf{W}$ matrix, usually the influence of other regions decreases with distance. This is represented by the product of $\mathbf{Z}$ and the inverse spatial transformation matrix $(\mathbf{I} - \gamma\mathbf{W})^{-1}$ in (3). Additionally, growth in each region is influenced by random shocks within the region, and by shocks coming from all the other regions $((\mathbf{I} - \gamma\mathbf{W})^{-1}\boldsymbol{\varepsilon})$, but once again, with an effect that usually decays with distance. Finally, it should be noticed that the spatial lag model given in (3) imposes an important constraint in the structure of spatial externalities: the spatial transformation, and thus the mechanism of spatial diffusion, is exactly the same in both $\mathbf{Z}$ and $\boldsymbol{\varepsilon}$.

The typical expression for the spatial error convergence equation can be written as:

$$\begin{aligned} \mathbf{g}_{\mathrm{y}} &= \mathbf{c} - \left(1 - \mathrm{e}^{-\beta T}\right)\ln\left(\mathbf{y}_{\mathbf{0}}\right) + \mathbf{X}\boldsymbol{\delta} + \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} = \lambda\mathbf{W}\boldsymbol{\varepsilon} + \boldsymbol{\nu} \\ \mathbf{g}_{\mathrm{y}} &= \mathbf{c} - \left(1 - \mathrm{e}^{-\beta T}\right)\ln\left(\mathbf{y}_{\mathbf{0}}\right) + \mathbf{X}\boldsymbol{\delta} + (\mathbf{I} - \lambda\mathbf{W})^{-1}\boldsymbol{\nu} \end{aligned} \tag{4}$$

In this case, it is evident that a random shock in a region affects growth rates in that region, and additionally impacts all the other regions through the spatial transformation. As a result, equation (4) recognises the presence of global externalities associated solely with random shocks.

The spatial error model in (4) can be expressed in the form of the spatial Durbin representation:

$$\begin{aligned}\mathbf{g}_{\mathrm{y}} = (\mathbf{I}-\lambda\mathbf{W})\mathbf{c} - (1-\mathrm{e}^{-\beta T})\ln(\mathbf{y}_{\mathbf{0}}) + \lambda\mathbf{W}\mathbf{g}_{\mathrm{y}} + \lambda(1-\mathrm{e}^{-\beta T})\mathbf{W}\ln(\mathbf{y}_{\mathbf{0}}) + \\ \mathbf{X}\boldsymbol{\delta} - \lambda\mathbf{W}\mathbf{X}\boldsymbol{\delta} + \boldsymbol{\nu}.\end{aligned} \tag{5}$$

The spatial Durbin explicitly shows the large number of parametric constraints that are involved in the spatial error model when conditioning variables are included in the growth equation. Relaxing the constraints results in a model with endogenous and exogenous spatial lags which nests the spatial lag and spatial error specifications.

Finally, the spatial cross-regressive model includes the spatial lag of initial income per capita as a right-hand-side variable:

$$\mathbf{g}_{\mathrm{y}} = \mathbf{c} - (1-\mathrm{e}^{-\beta T})\ln(\mathbf{y}_{\mathbf{0}}) + \mathbf{X}\boldsymbol{\delta} + \tau\mathbf{W}\ln(\mathbf{y}_{\mathbf{0}}) + \boldsymbol{\varepsilon} \tag{6}$$

As the effect of the spatial lag of income per capita is restricted to first-order neighbours, externalities are in, this case, local. This is also a special case of the unrestricted version of (5). In this case, the endogenous spatial lag is nullified and, the exogenous spatial lag is restricted to the initial income per capita.

It should be emphasised that most contributions have focused their attention on the spatial lag and the spatial error models, neglecting the spatial cross-regressive specification. This might be due to the non-significance of the coefficient for the spatial lag of initial income in some influential studies (see Fingleton 2003). Table 1 summarises the major characteristics, and the preferred specification, for a sample of studies that have included spatial dependence in the convergence equation.[3] Results from most of the studies favour the spatial error model against the spatial lag specification, that is, they support nuisance spatial dependence in the convergence equation. However, the presence of residual spatial dependence, and its modelling as a spatial error model, may reflect a more insidious cause. It may be that it is a manifestation of the omission of one or more spatially autocorrelated variables from matrix **X** in equation (1). After all, it is unlikely that such a simple model is likely to capture all of the actual causes of variation in productivity growth. Only five of the studies prefer the spatial lag model and its implication of substantive spatial dependence. Interestingly, with the exception of Fingleton (1999), the first group of studies excludes conditioning variables from the growth equation, while those preferring the spatial lag model include conditioning variables.

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[3] Studies included in this table are those that provide results for the spatial dependence tests leading to a choice between the spatial lag and the spatial error models. A comprehensive list of studies that have considered spatial effects in empirical models of growth can be found in Abreu et al. (2005).

Table 1. Studies that have included spatial effects in the convergence equation

Paper	Regions	Period	Spatial specification
Armstrong (1995)	EU NUTS I, II	1950–1990	Spatial Error (no X)
Fingleton and McCombie (1998,1999)	EU NUTS II	1979–1989	Spatial Error (no X)
Fingleton (1999)	EU NUTS II	1975–1995	Spatial Error (with X)
Rey and Montouri (1999)	US States	1929–1994	Spatial Error (no X)
Niebuhr (2001)	West Germany	1976–1996	Spatial Lag (with X)
Kosfeld et al. (2002)	Germany	1992–2000	Spatial Error (no X)
Vayá and Moreno (2002)	EU NUTS I, II	1975–1992	Spatial Error (no X)
Arbia et al. (2003)	Italy	1951–2000	Spatial Error (with and without Spatial regimes)
Lall and Shalizi (2003)	Brazilian Northeast municipalities	1985–1997	Spatial Lag & Error (with X)
Le Gallo et al. (2003)	EU NUTS I, II	1980–1995	Spatial Error (no X)
Ying (2003)	China	1978–1998	Spatial Lag (with X)
López-Bazo et al. (2004)	EU NUTS I, II	1975–1992	Spatial Error (no X), Spatial Lag (with X)
Dall’erba and Le Gallo (2005)	EU NUTS I, II	1989–1999	Spatial Lag (with X)

2.2 *Spatial dependence in the Verdoorn's Law*

In its simplest form, the empirical specification for Verdoorn's Law can be written as:

$$\mathbf{g}_y = \mathbf{c} + \frac{\kappa - 1}{\kappa}\mathbf{g}_Y + \boldsymbol{\varepsilon}, \tag{7}$$

where $\mathbf{g}_y$ and $\mathbf{g}_Y$ are growth in labour productivity and output of the manufacturing sector respectively, $\mathbf{c}$ is the intercept and $\boldsymbol{\varepsilon}$ a well-behaved error term. When $\kappa > 1$, the technology of production in manufactures is characterised by increasing returns to scale, and thus output grows more than proportionally with employment.

The spatial counterparts of (7) mimic those previously given for the convergence equation. Specifically, the spatial lag model is:

$$\begin{aligned} \mathbf{g}_y &= \mathbf{c} + \frac{\kappa - 1}{\kappa}\mathbf{g}_Y + \gamma\mathbf{W}\mathbf{g}_y + \boldsymbol{\varepsilon} \\ \mathbf{g}_y &= (\mathbf{I} - \gamma\mathbf{W})^{-1}\left(\mathbf{c} + \frac{\kappa - 1}{\kappa}\mathbf{g}_Y + \boldsymbol{\varepsilon}\right). \end{aligned} \tag{8}$$

The spatial error model is:

$$\begin{aligned} \mathbf{g}_y &= \mathbf{c} + \frac{\kappa - 1}{\kappa}\mathbf{g}_Y + \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} = \lambda\mathbf{W}\boldsymbol{\varepsilon} + \boldsymbol{\nu} \\ \mathbf{g}_y &= \mathbf{c} + \frac{\kappa - 1}{\kappa}\mathbf{g}_Y + (\mathbf{I} - \lambda\mathbf{W})^{-1}\boldsymbol{\nu}. \end{aligned} \tag{9}$$

The spatial Durbin representation becomes:

$$\mathbf{g}_y = (\mathbf{I} - \lambda\mathbf{W})\mathbf{c} + \frac{\kappa - 1}{\kappa}\mathbf{g}_Y - \lambda\frac{\kappa - 1}{\kappa}\mathbf{W}\mathbf{g}_Y + \lambda\mathbf{W}\mathbf{g}_y + \boldsymbol{\nu}. \tag{10}$$

Finally, the cross-regressive model is:

$$\mathbf{g}_y = \mathbf{c} + \frac{\kappa - 1}{\kappa}\mathbf{g}_Y + \tau\mathbf{W}\mathbf{g}_Y + \boldsymbol{\varepsilon}. \tag{11}$$

As for the empirical evidence, Table 2 summarises the contributions that have introduced spatial dependence as an additional element of Verdoorn's Law. Bernat (1996), for the US States, prefers the spatial error model, given the absence of spatial regimes; although given the presence of spatial regimes, the preferred model includes the spatial lag of labour productivity. Pons-Novell and Viladecans

Table 2. Studies that have included spatial effects in the Verdoorn's model

Paper	Regions	Period	Spatial Specification
Bernat (1996)	US States	1977–90	Spatial Error (no spatial regimes) Spatial Lag (with spatial regimes)
Fingleton and McCombie (1998)	EU NUTS II	1979–89	Spatial Lag (with X)
Pons-Novell and Viladecans (1999)	EU NUTS I	1984–92	Spatial Lag (no X)
Fingleton (2000)	EU NUTS II	1975–95	Spatial Lag (with X)
Fingleton and López-Bazo (2003)	EU NUTS II	1975–95	Spatial Lag (with X)
Fingleton (2004)	EU NUTS II	1975–95	Spatial Lag (with X)

(1999), replicating the same analysis for NUTS I EU regions also opt for the spatial lag model, and some previous studies show the same at the level of NUTS II EU regions. In some of these papers, no additional RHS variables are included, while in others they are.

Summing up, the empirical evidence on growth models with spatial dependence suggests that, especially when no additional variables are included in the list of regressors, the spatial error model is more often than not the chosen specification. In fact, the spatial error specification may be a catch-all for omitted spatially autocorrelated regressors. Nevertheless, the implication of the spatial error specification for the transmission of externalities is that they are essentially transmitted as random shocks. This is contrary to our hypothesis that these spatial externalities are essentially a substantive phenomenon. In other words, we prefer to treat them as effects with explicit and defined causes that can be modelled.

3 Growth externalities and substantive spatial dependence

In this section, we show how two growth models with across-region externalities due to knowledge diffusion can aid our understanding of why the straightforward application of spatial econometrics tools is likely to suggest inappropriate empirical growth specifications. The first model, that of López-Bazo et al. (2004), has its basis in neoclassical economic growth theory. The second model, given by Fingleton (2001 and 2004), is motivated by Verdoorn's Law, and relates to the theory underpinning New Economic Geography.

3.1 Substantive spatial externalities in the convergence equation

López-Bazo et al. (2004) start from a simple economy in which average labour productivity in region i in period t, y_{it}, is a function of the average level of physical and human capital per unit of labour, k_{it} and h_{it}, and the state of technology, A_{it}:

$$y_{it} = A_{it} k_{it}^{\tau_k} h_{it}^{\tau_h} \tag{11}$$

where τ_k and τ_h are internal returns to physical and human capital respectively.[4]

Technology in a region, A_{it}, is assumed to depend on the technological level of the neighbours, which in turn is related to their stock of both types of capital:

$$A_{it} = \Delta_t \left(k_{\rho it}^{\tau_k} h_{\rho it}^{\tau_h} \right)^{\gamma}, \tag{12}$$

where Δ_t is an exogenous component with a growth rate equal to g ($\Delta_t = \Delta_0 \mathrm{e}^{gt}$), $k_{\rho it}$ and $h_{\rho it}$ denote the physical and human capital-labour ratios in the neighbouring economies, and γ measures the externality across economies that is assumed to be positive: when $k_{\rho it}$ ($h_{\rho it}$) increases by 1%, causing an increase in the technology of those regions, technology in region i goes up by $\gamma\tau_k$ % ($\gamma\tau_h$ %).

Under such a technology of production, the steady state level of output per unit of effective labour ($\tilde{y}^*$) in any given region will depend positively on the stock of physical and human capital per unit of effective labour ($\tilde{k}$, $\tilde{h}$) in neighbouring regions in the case of positive spatial externalities ($\gamma > 0$):[5]

$$\tilde{y}^* = \left(\frac{s_k^{\tau_k} s_h^{\tau_h} \tilde{k}_{\rho}^{\gamma\tau_k} \tilde{k}_{\rho}^{\gamma\tau_h}}{(n+g+d)^{\tau_k + \tau_h}} \right)^{\frac{1}{1-\tau_k-\tau_h}} \tag{13}$$

where n, g and d denote population growth, the rate of technical progress and the depreciation rate, and s_k and s_h the rates of accumulation of physical and human capital. Meanwhile, the dynamics close to the steady state are characterised by the following growth equation (see the Appendix):

$$\begin{gathered} g_y = \xi - (1-\mathrm{e}^{-\beta T}) \ln(y_0) + \frac{(1-\mathrm{e}^{-\beta T})\gamma}{1-(\tau_k+\tau_h)} \ln(y_{0\rho}) + \gamma g_{y_\rho} + \\ \frac{(1-\mathrm{e}^{-\beta T})}{1-(\tau_k+\tau_h)} [\tau_k(\ln(s_k) - \ln(n+g+d)) + \tau_h(\ln(s_h) - \ln(n+g+d))] \end{gathered} \tag{14}$$

where $\beta = (1 - \tau_k - \tau_h)(n + g + d)$ is the rate of convergence, and:

$$\xi = (1+\gamma) g - (1-\mathrm{e}^{-\beta T}) \left(1 - \frac{\gamma}{1-(\tau_k+\tau_h)} \right) (\ln(\Delta_0) + gT).$$

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[4] These returns are the sum of the average firm's internal returns and intra-regional externalities in capital accumulation.

[5] Subscripts for regions and time periods are omitted to simplify notation. ρ is used to refer to neighbouring regions.

The empirical counterpart of (14) can be expressed as:

$$\mathbf{g}_{\mathrm{y}} = \mathbf{c} - (1 - \mathrm{e}^{-\beta T})\ln(\mathbf{y}_0) + \mathbf{X}\boldsymbol{\delta} + \phi_{\mathrm{Wy}}\ln(\mathbf{W}\mathbf{y}_0) + \gamma\mathbf{W}\mathbf{g}_{\mathrm{y}} + \boldsymbol{\varepsilon}. \tag{15}$$

This expression clearly indicates that both growth and initial income in 'neighbouring' economies matter for regional growth, with the across-economy externalities caused by knowledge diffusion inducing substantive spatial dependence in the convergence equation. It can also explain why the empirical evidence based on the traditional spatial model selection procedure has shown preference for the spatial error specification. Basically this is because of the similarity between equation (15) and the Durbin representation of the spatial error model when no conditioning X variables are included, as defined by equation (16).

The spatial Durbin representation reads as:

$$\mathbf{g}_{\mathrm{y}} = (\mathbf{I} - \lambda\mathbf{W})\mathbf{c} - (1 - \mathrm{e}^{-\beta T})\ln(\mathbf{y}_0) + \lambda\mathbf{W}\mathbf{g}_{\mathrm{y}} + \lambda(1 - \mathrm{e}^{-\beta T})\mathbf{W}\ln(\mathbf{y}_0) + \mathbf{u}, \tag{16}$$

whereas our model is given by:

$$\mathbf{g}_{\mathrm{y}} = \mathbf{c} - (1 - \mathrm{e}^{-\beta T})\ln(\mathbf{y}_0) + \gamma\mathbf{W}\mathbf{g}_{\mathrm{y}} + \gamma(1 - \mathrm{e}^{-\beta T})\ln(\mathbf{W}\mathbf{y}_0) + \mathbf{u}.$$

However, when control variables are included, both expressions clearly differ (more so with a higher number of conditioning variables).

The spatial Durbin representation reads as:

$$\begin{aligned}\mathbf{g}_{\mathrm{y}} = {} & (\mathbf{I} - \lambda\mathbf{W})\mathbf{c} - (1 - \mathrm{e}^{-\beta T})\ln(\mathbf{y}_0) + \lambda\mathbf{W}\mathbf{g}_{\mathrm{y}} + \lambda(1 - \mathrm{e}^{-\beta T})\mathbf{W}\ln(\mathbf{y}_0) + \\ & \mathbf{X}\boldsymbol{\delta} - \lambda\mathbf{W}\mathbf{X}\boldsymbol{\delta} + \mathbf{u},\end{aligned} \tag{17}$$

whereas our model is given by:

$$\mathbf{g}_{\mathrm{y}} = \mathbf{c} - (1 - \mathrm{e}^{-\beta T})\ln(\mathbf{y}_0) + \gamma\mathbf{W}\mathbf{g}_{\mathrm{y}} + \gamma(1 - \mathrm{e}^{-\beta T})\ln(\mathbf{W}\mathbf{y}_0) + \mathbf{X}\boldsymbol{\delta} + \mathbf{u}.$$

3.2 Substantive spatial externalities and Verdoorn's Law

Building on more or less the same theory that underpins some urban economic models and the new economic geography, Fingleton (2001 and 2004) derives an expression similar to the static Verdoorn Law,[6] relating the level of output per worker in the competitive sector (**y**) to the competitive sector output level (**Y**) and to the rate of technical progress ($\boldsymbol{\eta}$):

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[6] This is usually written in terms of the manufacturing sector.

$$\ln(\mathbf{y_t}) = \frac{\ln(\boldsymbol{\varphi})}{\kappa} + \left[\frac{\kappa - 1}{\kappa}\right] \ln(\mathbf{Y_t}) - \ln(\boldsymbol{\psi}) + \ln(\Delta_0) + \boldsymbol{\eta} t \tag{18}$$

where κ is the measure of returns to scale,[7] Δ_0 the initial level of technology, t is time, and $\boldsymbol{\varphi}$ and $\boldsymbol{\psi}$ parameters deriving from the technology of competitive sector production. The rate of technical progress is assumed to depend on three groups of variables: human capital (**H**), the initial level of technology (**G**) and the spillover of knowledge across regional boundaries ($\mathbf{W}\boldsymbol{\eta}$):

$$\begin{aligned} \boldsymbol{\eta} &= \mu \mathbf{H} + \pi \mathbf{G} + \gamma \mathbf{W} \boldsymbol{\eta} + \boldsymbol{\xi} \\ \boldsymbol{\eta} &= (\mathbf{I} - \gamma \mathbf{W})^{-1} (\mu \mathbf{H} + \pi \mathbf{G} + \boldsymbol{\xi}) \end{aligned} \tag{19}$$

where an autonomous rate, $\boldsymbol{\xi}$, is added since it is assumed that technical progress will occur as a result of learning by doing, irrespective of the other factors.

Inserting (19) into (18) and differentiating with respect to time, and inserting a well-behaved disturbance, $\boldsymbol{\varepsilon}$, results in:

$$\begin{aligned} \mathbf{g}_\mathrm{y} &= \frac{\kappa - 1}{\kappa} \mathbf{g}_\mathrm{Y} + (\mathbf{I} - \gamma \mathbf{W})^{-1} (\mu \mathbf{H} + \pi \mathbf{G} + \boldsymbol{\xi}) + \boldsymbol{\varepsilon} \\ \mathbf{g}_\mathrm{y} &= \frac{\kappa - 1}{\kappa} \mathbf{g}_\mathrm{Y} - \gamma \frac{\kappa - 1}{\kappa} \mathbf{W} \mathbf{g}_\mathrm{Y} + \gamma \mathbf{W} \mathbf{g}_\mathrm{y} + \mu \mathbf{H} + \pi \mathbf{G} + \boldsymbol{\xi} + (\mathbf{I} - \gamma \mathbf{W}) \boldsymbol{\varepsilon} \end{aligned} \tag{20}$$

where $\mathbf{g}_\mathrm{y}$ and $\mathbf{g}_\mathrm{Y}$ are growth in labour productivity and output, respectively.

As in the case of the convergence equation in (15), derived from a growth model with externalities across regions, the expression in (20) includes the spatial lag of growth of labour productivity, and of growth of output. Thus, once again, knowledge diffusion across regional boundaries provokes substantive spatial dependence in a Verdoorn-like growth specification. The equivalence of the Durbin representation of the spatial error model and the specification in (20) when the human capital and the technology gap are omitted, is again obvious in this case. This suggests that choosing the spatial error model in preference to substantive spatial dependence in the Verdoorn specification might be erroneous, and mostly caused by misspecification due to the omission of factors determining the rate of technical progress.

4 Empirical evidence

The aim of this section is twofold. First, we illustrate, for the case of EU regions, how the traditional spatial model selection procedure might not provide robust evidence on the preferred type of spatial dependence, and thus might lead to

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[7] This can be shown to equal the elasticity of competitive output with respect to the density of labour efficiency units.

erroneous conclusions about the kind of external effects across regions. Second, we show estimates of the growth specifications discussed in the previous section. Strong conclusions can be drawn about the existence and strength of spatial externalities because in this case specifications are built on a structural model of growth with knowledge diffusion across regions. We present the empirical evidence for both the convergence equation and Verdoorn's Law.

4.1 Convergence equation

Here we summarise some of the results obtained in López-Bazo et al. (2004) to show that the spatial error model is preferred to the spatial lag specification only when the analysis is based on the absolute β-convergence equation, that is, when no conditioning variables are included in the regression. In contrast, when we test for conditional β-convergence, the evidence supports substantive spatial dependence. The sample, and the variables used in the analysis, are described in detail in the above-mentioned reference. It is a sample of 108 EU regions for 12 initial EU countries, with data coming from the REGIO database maintained by EUROSTAT, the Statistical Office of the EU. The variable under analysis is the logarithmic rate of growth for gross domestic product per worker as a proxy for labour productivity in the period 1980–1996.

The lack of data for physical and human capital accumulation, and the effective rates of depreciation at the regional level in the EU, prevents us from using these variables when estimating the conditional β-convergence specification. Instead, we include additional explanatory variables to account for likely differences in accumulation and depreciation rates, technical change and any other factors influencing the level of technology across EU regions. To control for industrial mix, the shares of employment in agriculture, energy, manufacturing and construction are incorporated as additional covariates. We also include (the log of) an index of market potential to account for the direct impact of economic geography in the growth process. Following Harris (1954), market potential is defined as the sum of purchasing power of all other regions weighted by the inverse distance, to capture the effect of transport costs.

A measure of within-region innovative activity is also included. It is constructed as (the log of) the ratio of the number of patent applications to GDP. This can be considered a proxy for the output of technological activity in each region. Finally, (the log of) the yearly average temperature is added to the list of additional regressors, with the aim of capturing the effect of, for instance, social and cultural differences across EU regions.

Table 3 shows the results for the spatial dependence tests (Anselin et al. 1996), and the COMFAC test (Burridge, 1981).[8] As stated above, the tests have been used to select the preferred specification for spatial dependence. Results were obtained

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[8] This is a Likelihood Ratio test (or alternatively, a Wald test) on the set of non-linear constraints implied by the spatial Durbin representation of the spatial error model in (5). Let ϕ be the vector of coefficients for the explanatory variables, θ the vector for the spatially lagged explanatory variables and λ the spatial coefficient. Then, the null hypothesis can be expressed as H_0: $\lambda \cdot \phi = \theta$.

Table 3. Tests for spatial dependence in the convergence equation

Control vars	No	Yes
I-Moran	10.896***	8.276***
LM-ERR	93.311***	41.948***
Robust LM-ERR	23.456***	1.007
LM-LAG	69.864***	51.411***
Robust LM-LAG	0.0096	10.470***
LR-COMFAC	0.128	17.641**

Results have been obtained for a distance based weight matrix. ** and *** denote significant at 5% and 1%.

for a weight matrix based on the inverse of the square distance, although the test results are robust to some variations in the specification of the **W** matrix. In the case of the absolute convergence specification (first column of Table 3), results for the spatial dependence test suggest strong spatial dependence, and clearly point to the spatial error model as the favourite specification (the robust version of the Lagrange Multiplier error test rejects its null hypothesis of no spatial dependence, while the test for the spatial lag does not). In addition, the COMFAC test does not reject the null hypothesis, and hence the parametric constraints in the spatial Durbin representation in (16) are not rejected in the sample of EU regions.

The conclusion completely changes when conditioning variables are included in the convergence equation. In this case, results for the spatial autocorrelation tests in column 2 of Table 3 clearly indicate that there is substantive spatial dependence (the robust spatial lag test rejects the null hypothesis while the robust error test does not). Accordingly, the COMFAC test rejects the parametric constraints in the Durbin representation in (17). As indicated in the previous section, close similarity between the empirical specification derived from the theoretical model and the Durbin representation only exists when there are no control variables present in the model to account for differences in the steady-states. When control variables are present, the theoretical model and the Durbin representation become quite dissimilar.

In order to demonstrate that spatial externalities exist and to evaluate their strength, we estimate the structural growth model given as equation (15). Table 4 summarises the results obtained by excluding and including the conditioning variables (first column and second column, respectively). For the purpose of maximum likelihood estimation, we assume that the explanatory variables are exogenous, so that the only endogenous right-hand-side variable in our analysis is the spatial lag of labour productivity growth. The coefficient associated with knowledge diffusion across regions is statistically significant, positive and very large in magnitude. Thus, results for the sample of EU regions support our thesis regarding the importance of externalities that cross regional boundaries in the production process. It should be noticed that these results are robust to the inclusion of conditioning variables in the estimated model. Additionally, the LM-ERR test indicates there is no significant evidence (using the conventional Type I error

Table 4. Estimation of the growth equation with externalities across economies

γ	0.893***	0.884***
β	0.024***	0.030***
ϕ_{Wy}	0.281***	0.237*
Control var	No	Yes
LnL	72.94	78.48
AIC	−137.87	−134.95
LR-LAG	55.871***	46.369***
LM-ERR	2.63	3.195*

*, **, *** means significant at 10%, 5% and 1%.

Table 5. Tests for spatial dependence in the Verdoorn's model

Control vars	No	Yes
I-Moran	6.817***	5.891***
LM-ERR	30.899***	20.092***
Robust LM-ERR	7.231**	2.460
LM-LAG	23.678***	35.036***
Robust LM-LAG	0.0104	17.403***
LR-COMFAC	0.313	10.341*

***, **, *: means significant at 1%, 5% and 10%. From Table 19.2 and Table A4 in Fingleton (2004) and own calculations.

rate of 0.05) of remaining spatial dependence in the growth equation once we include externalities across regions. Of course, these estimates are to some extent conditional, an issue we briefly consider after first discussing the results of the Verdoorn specification.

4.2 The Verdoorn law specification

The evidence that is used to illustrate the issue of spatial effects under the Verdoorn law specification is taken from Fingleton (2004) and our own calculations. In this case, the data are for 178 EU regions over the period 1975–1995, and focus on productivity growth in the manufacturing sector, which is taken to equate to the competitive sector in this analysis. The weight matrix takes into account the size of each regional economy, measured by its output, and the square of the inverse of the distance between each pair of regions ($Q_j d_{ij}^{-2}$). The share of the population aged 25–59 with higher educational attainment levels is used as a proxy for human capital (**H**), while the start-of-the-period level of manufacturing technology gap (**G**) is defined as one minus the ratio of the region's manufacturing productivity level in 1975 to that of the leading region. Table 5 reproduces and extends the results of the spatial dependence tests for the case of the expanded Verdoorn's specification in Fingleton (2004). There is strong evidence of spatial

Table 6. Estimation of the Verdoorn's equation with technological diffusion

γ	0.6920***	0.5321***
$(\kappa - 1)/\kappa$	0.5863***	0.5183***
π	–	0.0391***
μ	–	0.0434***
$\gamma(\kappa - 1)/\kappa$	–0.4057	–0.2758
Control vars	No	Yes
LnL	497.9551	514.2192
AIC	–989.91	–1018.4384
LR-LAG	24.609***	15.433***
LM-ERR	0.5886	1.361

ML estimation. Similar results are obtained by using iterated two-stage least squares based on the approach outlined in Fingleton (2004). *, **, *** means significant at 10%, 5% and 1%.

autocorrelation in the non-spatial Verdoorn equation. As in the case of the conditional convergence equation, the preferred specification with additional regressors is the spatial lag (only the robust LM-LAG rejects the null hypothesis, and constraints on the parameters of the Durbin representation are rejected at the 5% level, as indicated by the COMFAC test). Notice that equation (20) incorporates a moving average error process $(\mathbf{I} - \gamma\mathbf{W})\boldsymbol{\varepsilon}$ which is ignored in estimation. However, there is no evidence of residual spatial autocorrelation that might be caused by this.

Estimation of the coefficients of the structural model (Table 6) provides strong support for the hypothesis that there exists a high rate of technological diffusion across regions. More than 50% of technical progress generated in a representative EU region diffuses to its 'neighbours'.[9] In addition, the inclusion of technological diffusion in the growth equation completely accounts for spatial dependence in the manufacturing productivity growth rates.

As with the neoclassical growth analysis in Tables 3 and 4, the inferences about technical diffusion are conditional on the structure of the weights matrix, although the results we obtain are quite robust to variation of the assumed interaction between regions as embodied within the weights matrix. Also, for maximum likelihood estimation, the assumption is that the regressors are exogenous. This might be questioned in the context of the Verdoorn equation, since manufacturing output growth may not only be a cause of productivity growth, but also depend on it. Likewise, our measure of human capital may depend on manufacturing productivity growth. In addition, the presence of heterogeneity may affect the estimates obtained.

These issues have been considered in some detail in the literature (Fingleton 2000, 2004; Fingleton and López-Bazo 2003; Bivand and Brunstad 2006), and alternative estimates were produced taking account of these various factors. However, these are quite similar to what we choose to give here as an illustration. For simplicity and clarity, we simply give the results of maximum likelihood

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[9] Notice that the control variables in Table 5 are not the same as those used in Table 6.

estimation, since none of the issues raised above substantially change the estimates obtained. The conclusions we arrive at are reinforced, rather than undermined, by the additional evidence present in the published literature. It is true that, in general, empirical modelling is also conditional on the area of study, the level of spatial aggregation (here NUTS 2 regions of the EU) and the time period adopted for study, and therefore it would be interesting to explore the extent to which our estimates and conclusions are robust to alternative data sets. However, this is beyond the scope of this article.

Conclusions

There has been a remarkable surge of interest in 'geographical economics' or 'the new economic geography', prompted by the publication of the book by Fujita et al. (1999). This new wave of theory put economic geography on the forefront of mainstream economics, since it established the notion that increasing returns could coexist within a theoretical framework with explicit microeconomic foundations. Regional science and regional economics, which tended to be somewhat marginalised, have now become a focus of attention. However, the development of formal models has been at a cost, for although the idea of externalities is central to the new economic geography theory, and related urban economic theory (Abdel-Rahman and Fujita 1990; Rivera-Batiz 1988), in the purest form of these models, the only externalities present are pecuniary externalities, which represent market interdependence. The idea that technological externalities are also relevant is somehow squeezed out, being too difficult to accommodate within formal models.

Nonetheless, many geographical economists have attempted to capture both pecuniary and technological external economies in their empirical models, reflecting their broader emphasis both on theoretical consistency and empirical veracity. This is particularly the case when regional economists have applied spatial econometric models, fitting these models to real data. Without also controlling for externalities in the form of spillovers between regions, the models are invariably poorly specified and fail the diagnostic tests conforming to accepted professional standards of the spatial econometrics community. Various approaches have been adopted in attempting to introduce externalities into spatial econometric models, with two main strands appearing in the literature. One treats the externalities in a somewhat ad hoc manner. In these models, there is no attempt to explicitly model the sources of these external effects. The second strand attempts to model the causes of the externalities. This article argues that there is good reason to favour this second approach, although it may be more demanding in terms of data. Looking at some of the literature, we find that it usually is the spatial error model that is preferred on the basis of simple specifications that are devoid of conditioning variables. The external effects are simply treated as nuisance variables. In the case where conditioning variables are present, such as in neoclassically-oriented conditional convergence models of economic growth or in enhanced Verdoorn-like models with a basis in the new economic geography and urban economics, it is frequently models with an explicit representation of the spillover process that are chosen. Often these models have exogenous spatial lags, or perhaps an endogenous spatial lag, or both, thus representing spillovers as substantive rather than nuisance effects.

Our preference at this point in time is for this type of explicit externality modelling, since it is our understanding that the selection of the spatial error model is oftentimes based on diagnostic indicators that reflect the existence of omitted effects that should, if possible, be included as important and explicit variables in our modelling. However, while we have focused on substantive versus nuisance representations of spillover effects, it is also possible that both could be present at the same time as real phenomena. Although we do not find any significant evidence for error dependence in the presence of the endogenous spatial lag, it would be interesting to estimate models in which both are present. This could be accomplished via GMM (Kelejian and Prucha 1998; Badinger and Tondl 2003), and remains a task for the future.

Appendix

The derivation of the growth equation describing transitional dynamics is standard in the growth literature (Mankiw et al. 1992; Barro and Sala-i-Martin 1992). In this Appendix, we derive the growth equation given in (14), where externalities across economies are included in the technology of production. Subscripts *i* and *t* are omitted to ease notation.

By substituting (12) in (11), and writing the variables in units of effective labour ($\tilde{y} = Y/\Delta L$, $\tilde{k} = K/\Delta L$, and $\tilde{h} = H/\Delta L$),

$$\tilde{y} = \tilde{k}^{\tau_k} \tilde{h}^{\tau_h} \left(\tilde{k}_\rho^{\tau_k} \tilde{h}_\rho^{\tau_h} \right)^{\gamma} \tag{A.1}$$

Growth for product by effective labour ($g_{\tilde{y}}$) is:

$$g_{\tilde{y}} = \tau_k g_{\tilde{k}} + \tau_h g_{\tilde{h}} + \gamma \left(\tau_k g_{\tilde{k}_\rho} + \tau_h g_{\tilde{h}_\rho} \right) \tag{A.2}$$

where $g_{\tilde{k}}$ and $g_{\tilde{h}}$ are the law of motion for the accumulation of $\tilde{k}$ and $\tilde{h}$. Using the first order Taylor expansion around the steady state for their expressions, they can be written as:

$$\begin{aligned} g_{\tilde{k}} &= \frac{\dot{\tilde{k}}}{\tilde{k}} = (\tau_k - 1)(n+g+d)\left(\ln \tilde{k} - \ln \tilde{k}^*\right) + \tau_h (n+g+d)\left(\ln \tilde{h} - \ln \tilde{h}^*\right) \\ g_{\tilde{h}} &= \frac{\dot{\tilde{h}}}{\tilde{h}} = \tau_k (n+g+d)\left(\ln \tilde{k} - \ln \tilde{k}^*\right) + (\tau_h - 1)(n+g+d)\left(\ln \tilde{h} - \ln \tilde{h}^*\right). \end{aligned} \tag{A.3}$$

Substituting (A.3) in (A.2), and defining the rate of convergence as:
$\beta = (1 - \tau_k - \tau_h)(n + g + d)$, $g_{\tilde{y}}$ can be expressed as:

$$g_{\tilde{y}} = -\beta\left(\tau_k \ln \tilde{k} + \tau_h \ln \tilde{h}\right) + \beta\left(\tau_k \ln \tilde{k}^* + \tau_h \ln \tilde{h}^*\right) + \gamma\left(\tau_k g_{\tilde{k}_\rho} + \tau_h g_{\tilde{h}_\rho}\right) \qquad \text{(A.4)}$$

Combining (A.4) with expressions for the steady state for $\tilde{k}$ and $\tilde{h}$, under the assumption of decreasing returns to capital within each region, $(\tau_k + \tau_h) < 1$,

$$\begin{aligned} \tilde{k}^* &= \left(\frac{s_k^{1-\tau_h} s_h^{\tau_h} \tilde{k}_\rho^{y\tau_k} \tilde{h}_\rho^{y\tau_h}}{n+g+d} \right)^{\frac{1}{1-\tau_k-\tau_h}} \\ \tilde{h}^* &= \left(\frac{s_k^{\tau_k} s_h^{1-\tau_k} \tilde{k}_\rho^{y\tau_k} \tilde{k}_\rho^{y\tau_h}}{n+g+d} \right)^{\frac{1}{1-\tau_k-\tau_h}} \end{aligned} \qquad \text{(A.5)}$$

we obtain,

$$\begin{aligned} g_{\tilde{y}} = {} & -\beta \ln \tilde{y} + \frac{\beta\gamma}{1-\tau_k-\tau_h}\left(\tau_k \ln \tilde{k}_\rho + \tau_h \ln \tilde{h}_\rho\right) + \\ & \frac{\beta}{1-\tau_k-\tau_h}(\tau_k \ln s_k + \tau_h \ln s_h - (\tau_k + \tau_h)\ln(n+g+d)) + \gamma\left(\tau_k g_{\tilde{k}_\rho} + \tau_h g_{\tilde{h}_\rho}\right) \end{aligned} \qquad \text{(A.6)}$$

Under the assumption that there is no region large enough to influence productivity for all its neighbours by itself:

$$\tau_k \ln \tilde{k}_\rho + \tau_h \ln \tilde{h}_\rho \cong \ln \tilde{y}_\rho \qquad \text{(A.7)}$$

Thus, substituting (A.7) in the previous expression, we obtain:

$$\begin{aligned} g_{\tilde{y}} = {} & -\beta \ln \tilde{y} + \frac{\beta\gamma}{1-\tau_k-\tau_h} \ln \tilde{y}_\rho + \gamma g_{\tilde{y}_\rho} + \\ & \frac{\beta}{1-\tau_k-\tau_h}(\tau_k \ln s_k + \tau_h \ln s_h - (\tau_k + \tau_h)\ln(n+g+d)). \end{aligned} \qquad \text{(A.8)}$$

Finally, from (A.8) growth for product by effective labour between periods 0 and T can be expressed as:

$$g_{\tilde{y}} = -(1-\mathrm{e}^{-\beta T})\ln\tilde{y}_0 + \frac{(1-\mathrm{e}^{-\beta T})\gamma}{1-(\tau_k+\tau_h)}\ln\tilde{y}_{0\rho} + \gamma g_{\tilde{y}_\rho} + \frac{(1-\mathrm{e}^{-\beta T})}{1-(\tau_k+\tau_h)}[\tau_k(\ln s_k - \ln(n+g+d)) + \tau_h(\ln s_h - \ln(n+g+d))]. \quad \text{(A.9)}$$

Or, equivalently, the growth equation in terms of product by labour corresponding to equation (14).

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